ARGONNE NATIONAL LABORATORY
9700 South Cass Avenue
Argonne, Illinois 60439

Propagation of data error and parametric sensitivity in computable general equilibrium models

Joshua Elliott, Meredith Franklin, Ian Foster, Margaret Loudermilk, and Todd Munson

Mathematics and Computer Science Division

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Contents

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Joshua Elliott† Meredith Franklin‡ Ian Foster§ Margaret Loudermilk¶
Todd Munson‖

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Abstract

While computable general equilibrium (CGE) models are a well-established tool in economic analyses, it is often difficult to disentangle the effects of policies of interest from that of the assumptions made regarding the underlying calibration data and model parameters. To characterize the behavior of a CGE model of carbon output with respect to two of these assumptions, we perform a large-scale Monte Carlo experiment to examine its sensitivity to base year calibration data and elasticity of substitution parameters in the absence of a policy change. By examining a variety of output variables at different levels of economic and geographic aggregation, we assess how these forms of uncertainty impact the conclusions that can be drawn from the model simulations. We find greater sensitivity to uncertainty in the elasticity of substitution parameters than to uncertainty in the base-year data as the projection period increases. While many model simulations were conducted to generate large output samples, we find that few are required to capture the mean model response of the variables tested. However, characterizing standard errors and empirical probability distribution functions is not possible without a large number of simulations.

1 Introduction

Computable general equilibrium (CGE) models aim to quantify the effects of policies on equilibrium allocations and relative prices using standard general equilibrium theory. Interest in such models has grown since their advent in the 1960s as applied economists have recognized the benefits of using them for counterfactual analysis and as improvements in computation have allowed for greater detail. Today CGE modeling is an established methodology in applied economics and has become an indispensable tool for policy analysis. Since the 1990s, CGE models have been used extensively in environmental economics and in integrated assessment models (IAMs). IAMs couple economic and climate models to predict the environmental and economic impacts of climate change and to assess the effects of mitigation policies [5, 6, 8–10, 36, 38, 39].

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†Computation Institute, U. Chicago and Argonne National Laboratory, email: jelliott@ci.uchicago.edu
‡U. Chicago and Argonne National Laboratory, email: mfranklin@anl.gov
§Computation Institute, U. Chicago and Argonne National Laboratory, email: foster@anl.gov
¶Computation Institute, U. Chicago and Argonne National Laboratory, email: mloudermilk@uchicago.edu
‖Computation Institute, U. Chicago and Argonne National Laboratory, email: tmunson@mcs.anl.gov

CGE models are often criticized for imposing behavioral assumptions without empirical support. These assumptions include the functional forms for consumer utility and firm production functions and their elasticity of substitution and share parameter values. Much research has been undertaken to combat this critique in the economics literature. Some studies compute econometric estimates for all model parameters from time series data [12, 26, 27, 32]. Many others employ base-year data in combination with exogenous parameter estimates to calibrate the model and assess the sensitivity of the model output to the base-year data [2, 13, 35] and to the exogenous parameter estimates [14, 21, 24, 33, 34, 41].

Most IAMs use CGE models with nested constant elasticity of substitution (CES) production and utility functions. While these models can have any number of parameters that encode model dynamics, such as labor productivity and energy efficiency, the CES functions are defined by the cost share parameters and the elasticities of substitution among commodities. Cost share parameters are derived from base-year data, while the elasticities of substitution are chosen by either expert opinion [40] or econometric estimates from time-series data [28]. All of these parameters are subject to uncertainty in their values, and the sensitivity of models to this uncertainty is often poorly understood.

The IAM community typically either studies the sensitivity of model output to only a small subset of parameters or uses only a highly simplified IAM for their analysis. For example, [40] explores model sensitivity to only the elasticities of substitution for capital, labor, and energy and assumes these elasticities vary by industry but remain constant across regions, while [25] evaluates sensitivity to only the Armington international trade elasticities. To our knowledge, a study of the sensitivity of model output to the cost share parameters derived from base-year expenditure data has never been undertaken, likely because of the large sample sizes necessary.

One important distinction in the way CGE models are used to evaluate climate policies between the IAM community and the applied economics community is in the metrics. In applied economics, a model is run with business-as-usual assumptions to produce a set of baseline measures and then "shocked" with a representation of a given policy. The impacts of the policy shock are evaluated *relative* to the hypothetical baseline levels. The magnitude and sensitivity of the baseline levels themselves, however, are important in IAMs. When forecasting future concentrations of atmospheric CO_2, for example, the explicit emissions trajectory is important due to the physical effects of global CO_2 concentration. The relative change in emissions to their baseline levels provides only part of the required information. Moreover, policy targets are explicitly measured relative to a historical level, such as the 17% reduction from 2005 levels by 2020 target, making it especially important if emissions in the baseline differ by a few percent in 2020, even if the emissions reduction from a given policy relative to the baseline level is not sensitive to the uncertainty.

In this study, three large sets of model simulations were conducted using the CIM-EARTH CGE model [17] to examine the sensitivity of the output to uncertainty in the share parameters and the elasticities of substitution. These simulations are motivated not by the need for characterizing the distributions of specific policy relevant metrics but rather by the relative lack of studies examining these particular sets of parameters. The analysis of these results exposes the level of "noise" inherent in the models. We have not attempted to characterize uncertainty in the dynamic parameters which depend strongly on the underlying assumptions of the modeler and can have very significant impacts on the outcome of a simulation. Since the parametrization of dynamics vary significantly among models, the information attained from such sensitivity studies would be difficult to generalize. Instead, this study evaluates measurables only under a set of simple baseline assumptions and examines two sets of parameters common to most CGE models.

The remainder of the paper proceeds as follows: Section 2 presents the CIM-EARTH CGE model, details the parameter distributions used for our analysis, and describes the statistical meth-

Table 1: Regions, industries, and factors for the CIM-EARTH CGE model. Regions are labeled with our estimate of the level of uncertainty in their economic data: (L) low, (M) medium, and (H) high. The industries are labeled by their production function structure: (A) agriculture, (E) extraction of fossil fuels, (M) manufacturing, (N) electricity generation, (P) petroleum refining, and (S) service industries.

Regions	**Industries**	**Factors**
Canada (L)	Agriculture and Forestry (A)	Capital
Mexico (M)	Coal Extraction (E)	Labor
United States (L)	Gas Extraction (E)	Land
Brazil (H)	Oil Extraction (E)	Natural Resources
Rest of Latin America (H)	Cement (M)	
Western Europe (L)	Chemicals (M)	
Rest of Europe (M)	Nonferrous Metals (M)	
Middle East and North Africa (M)	Steel and Iron (M)	
Rest of Africa (H)	Other Manufacturing (M)	
China, Mongolia, and Koreas (H)	Electricity (N)	
India (H)	Petroleum Refining (P)	
Japan (L)	Air Transport (S)	
Russia, Georgia, and Asiastan (M)	Land Transport (S)	
Rest of South Asia (H)	Sea Transport (S)	
Rest of Southeast Asia (M)	Government Services (S)	
Oceania (L)	Other Services (S)	

ods applied; Section 3 reports the sensitivity results obtained from our simulations; and Section 4 summarizes our findings.

2 Methodology

The CIM-EARTH CGE model used in this study is a myopic model with 16 regions and 60 time periods (2004–2063). Each region has 16 domestic industries, 16 importers, one capital goods industry, one consumer, and four production factors. There are also three homogeneous transportation service industries. The regions, industries, and factors in the model are found in Table 1. Each region is labeled with our evaluation of the level of uncertainty in the economic data: (L) low, (M) medium, and (H) high. For each industry we indicate the structure of the production functions: (A) agriculture, (E) extraction of fossil fuels, (M) manufacturing, (N) electricity generation, (P) petroleum refining, and (S) service industries.

Capital in this model is not mobile across regions, but is perfectly mobile across industries within a region and depreciates at 4% annually. To spur investment in capital, investment contributes to consumer utility with the investment amount calibrated to historical data. We employ exogenous trajectories for labor productivity and supply, energy efficiency, and resource availability based on the dynamic equations of the EPPA model [3]. Carbon emissions from fossil fuel usage are derived from the energy volume information in GTAP-E [11].

Nested CES functions are used to model both firm production functions and consumer utility.

The calibrated CES production functions [7] have the form

$$\mathbf{y} = \left(\sum_i \theta_i \mathbf{x}_i^{\frac{\sigma-1}{\sigma}} \right)^{\frac{\sigma}{\sigma-1}} ,$$

where $\mathbf{y}$ and $\mathbf{x}$ are the changes in output and inputs relative to a base year, respectively, and θ are the cost share parameters calibrated from base-year data with $\theta_i > 0$ and $\sum_i \theta_i = 1$. The elasticity of substitution parameter σ controls to what degree the inputs can be substituted for one another. When $\sigma = 0$, we obtain the Leontief production function

$$\mathbf{y} = \min_i \{\mathbf{x}_i\} ,$$

implying that the inputs are perfect complements. The calibrated cost share parameter is the ratio of the base-year industry expenditure on that commodity to the total industry revenue. The choice of cost share parameters ensures that the output and inputs are consistent with the base-year data.

These functions are then nested to constrain the decisions made by the industries and consumers, where the structure is represented by a tree. Each node in the tree represents a CES function with its own elasticity of substitution that aggregates the inputs from below into a commodity bundle. The root node aggregates the commodity bundles into the total industry output. Figure 1 shows the trees used for the domestic producers and the importers of each commodity in each region for model in this paper. The structure of the production functions for each industry type is loosely based on the EPPA model [3]. The capital goods industries aggregate materials using a single Leontief production function and do not demand fossil fuels, refined petroleum, electricity, or production factors. These capital goods are demanded only by consumers. The homogeneous transportation service industries simply aggregate air, land, and sea transportation services from each region into a single commodity using a Leontief production function. These homogeneous transportation services are used only for international trade.

The CIM-EARTH CGE model is coded in the AMPL modeling language [20] and solved by applying the PATH algorithm [15, 18, 19]. For complete details, see the CIM-EARTH CGE model documentation [17].

The predictions generated by this model are highly dependent on the choices of values for the share parameters and the elasticities of substitution, and thus on the data from which they are estimated. Since the share parameters are computed from the base-year cost data, we refer to uncertainty in this data as *cost share uncertainty*. We refer to uncertainty in the elasticity of substitution parameters as *elasticity uncertainty*. To make our studies tractable, we limit the number of uncertain parameters by applying simplifying assumptions. We now document our methodology for studying the sensitivity of variables to these uncertainties.

2.1 Cost Share Uncertainty

Since the cost share parameters are computed from base-year data, we can equivalently determine the sensitivity to uncertainty in the cost data. The CIM-EARTH CGE model uses the GTAP version 7 database for the 2004 base-year expenditures [23].[1] The primary source of error in this database is reporting error. Another source of error is the lack of updated data. For example, of

[1]The GTAP database is constructed by removing inconsistencies in the raw data supplied for each region using a balancing routine. Ideally, we would treat the raw cost data as uncertain and apply this balancing routine to samples drawn from the uncertain raw data. However, using this process was not possible because of restricted access to both the raw data and the balancing routine for GTAP and to the large number of values in the fully disaggregated database.

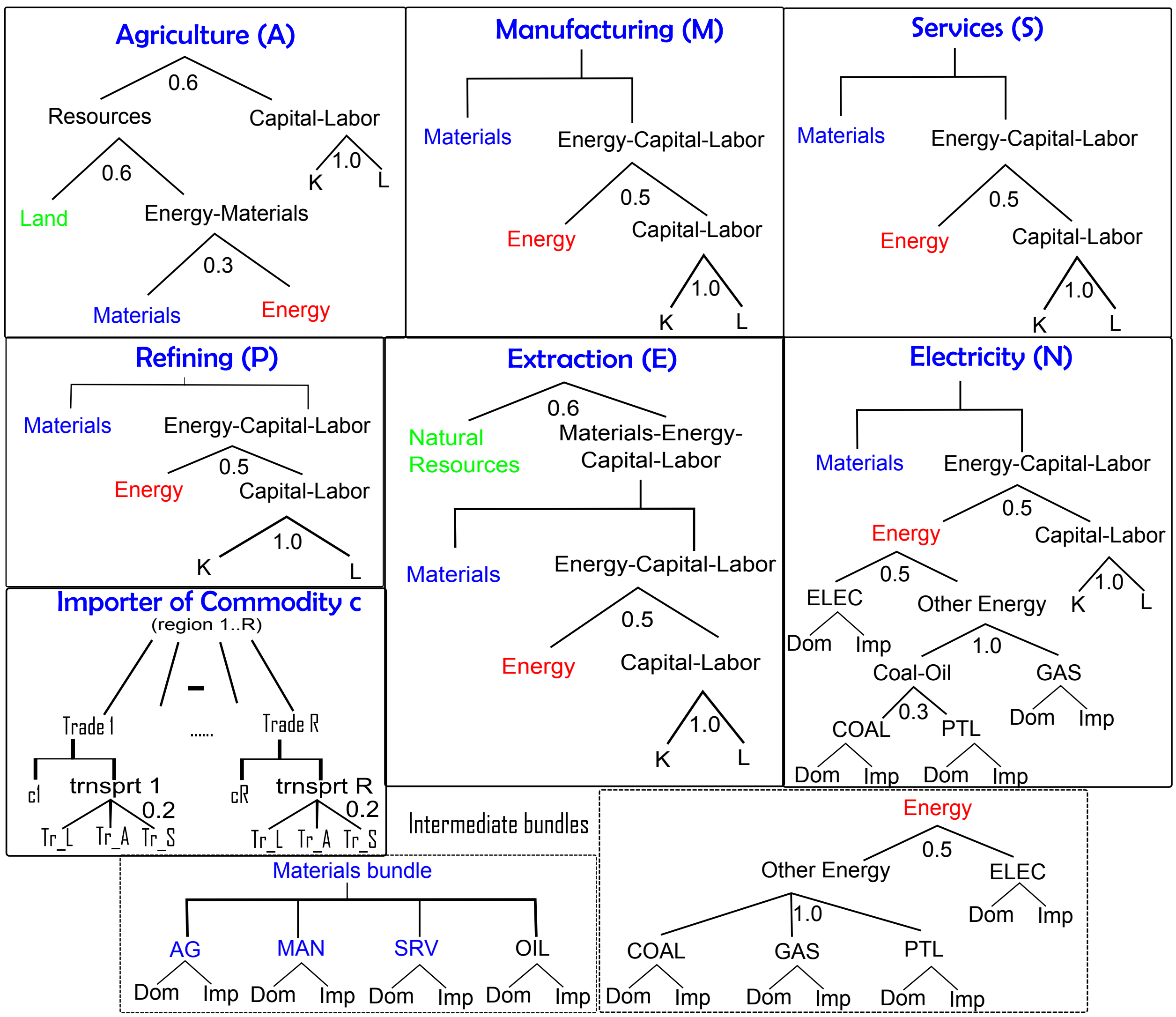

Figure 1: CIM-EARTH CGE nested CES production functions. Each node represents a CES function. Nodes with vertical line inputs represent Leontief functions. The other nodes are labeled with their elasticities of substitution.

the 87 regions and countries in the GTAP version 6 database with a 2001 base year, less than half reported new 2004 cost data for version 7. When new data is not reported, the 2001 cost data is scaled to obtain 2004 cost data. However, the 2001 data may in fact be a scaled version of 1997 data from GTAP version 5.

The full GTAP version 7 database has 113 regions (R), 57 industries (I), and 5 factors (F). The maximum number of cost share parameters in a particular $R \times (I + F)$ aggregation is $RI(2I + F)$ for industries, $RI(4R)$ for importers (including land, sea, and air transportation), and $R(2I + 1)$ expenditures by consumers (including demand for savings). Therefore, the full database has a maximum of approximately 3.7 million expenditure values. Many of these values, however, are zero. With 16 regions, 16 industries, and four factors, the CIM-EARTH CGE model has a maximum of approximately 26,000 expenditure values. Approximately 40% of these values are ignored because the amount is less than $1M. The remaining parameter space, however, is still large. For this study, we chose a parameter space consisting of the 100 largest expenditure values per region, with a maximum of 5 chosen from each industry, leading to 1,600 uncertain cost share parameters. While we restricted the analysis to this smaller set of values, they still account for more than 75% of global expenditures.

We use uncorrelated Gaussian distributions to characterize the uncertainty in the expenditure values.[2] The taxes and subsidies on those expenditures were chosen to maintain constant rates. The standard deviation of the distribution about the mean value is based on our estimate of the level of uncertainty in the economic data for each region. The regions indicated by (L) in Table 1 are believed to have well-established structures in place for consistent and accurate data gathering. Therefore, we assume the reported costs from these regions have low levels of uncertainty and set the standard deviation to 3% of the mean value. In contrast, we assume that poorly-developed regions and regions notorious for having data inconsistencies have high levels of uncertainty in their reported values. For these regions indicated by (H) in Table 1, we set the standard deviation of each distribution to 7% of the mean value. For all other regions indicated by (M) in Table 1, we assume medium levels of uncertainty and set the standard deviation of each distribution to 5% of the mean value. Unsurprisingly, the regions with low levels of uncertainty in their economic data have updated cost data for 2004 in the GTAP version 7 database. Most regions categorized as having a medium or high level of uncertainty in their economic data, including China, South and Southeast Asia, and most of Africa and Latin America, did not provide updated data for 2004. Rather, the 2004 data was obtained by scaling the 2001 data from GTAP version 6.

2.2 Elasticity Uncertainty

The elasticities of substitution in our model are based on parameters from the EPPA [37, 40] and GTAP [31] CGE model documentation and on recent estimates derived from historical U.S. Bureau of Economic Analysis data [4]. The elasticities of substitution for most commodity bundles are indicated in Figure 1. The elasticities of substitution between domestic and imported commodities and the Armington international trade elasticities are found in Table 2.

The elasticity of substitution parameters play a key role in the predictions generated by CGE models, yet there are vast inconsistencies in the estimates used by different models. The plot in Figure 2, for example, compares the estimates for the elasticity of substitution between capital and

[2] While some costs may be correlated, we did not account for this possibility in the uncertainty distributions, even though such correlations would have an effect on the sensitivity of the variables. Establishing the existence and extent of the correlation would require a more detailed examination of the underlying covariance structures. The actual tax and subsidy amounts and carbon emission factors may also be uncertain because of reporting errors or lack of updated data. We did not consider the sensitivity of variables to uncertainty in these values.

Table 2: Mean elasticity of substitution parameters between domestic and imported commodities and the Armington international trade elasticities by industry for the CIM-EARTH CGE model.

Industry	**Elasticity of Substitution** Domestic/Import	Armington
Agriculture and Forestry	2.7	5.6
Coal Extraction	3.0	6.1
Gas Extraction	17.2	34.4
Oil Extraction	5.2	10.4
Cement	2.9	5.8
Chemicals	3.3	6.6
Nonferrous Metals	4.2	8.4
Steel and Iron	3.0	5.9
Other Manufacturing	3.4	7.2
Electricity	2.8	5.6
Petroleum Refining	2.1	4.2
Air Transport	1.9	3.8
Land Transport	1.9	3.8
Sea Transport	1.9	3.8
Government Services	1.9	3.8
Other Services	1.9	3.8

labor in the coal extraction industry used by the EPPA [37] and GTAP [31] CGE models and the estimate from Balistreri et al. [4]. Moreover, when we look at the elasticity of substitution between capital and labor across many industries, we can discern no consistent pattern in the disagreement. The GTAP parameter, for example, is at times the lowest and at other times the highest. This analysis is summarized in Appendix A.

For this study, we follow previous studies [25, 37, 40] and do not allow the elasticities of substitution to vary by region because of both a lack of data to support this differentiation and a desire to reduce the number of uncertain parameters. Therefore, the production functions for each industry have the same structure and elasticity of substitution parameters independent of the region. Moreover, we assume that the Leontief nests in the production functions are certain. Our assessment of uncertainty therefore consists of evaluating model sensitivity to 70 elasticity of substitution parameters obtained by aggregating similar industries. The complete list of parameters is found below.

- 16 elasticity of substitution parameters between capital and labor
- 16 elasticity of substitution parameters between domestic and imported commodities
- 16 Armington international trade elasticities
- 5 additional elasticity of substitution parameters common across the (A) industries
- 4 additional elasticity of substitution parameters common across the (E) industries
- 3 additional elasticity of substitution parameters common across the (M) industries
- 4 additional elasticity of substitution parameters common across the (N) industries

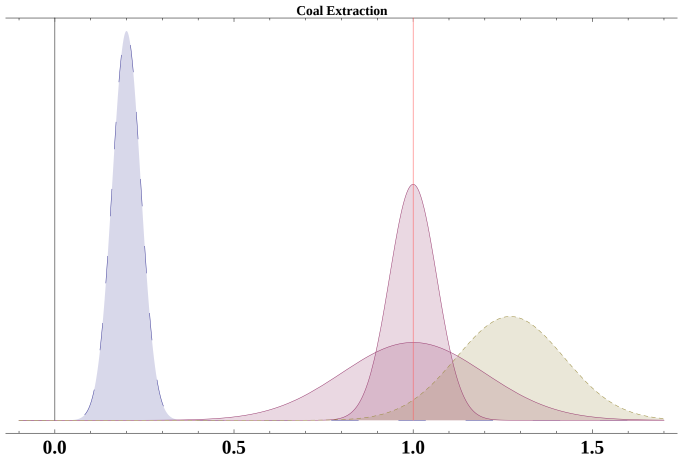

Figure 2: Comparison of distributions for the elasticity of substitution between capital and labor for the coal industry. Blue dashed: GTAP estimate with standard deviation set to 20% of the mean; yellow dotted: estimate from Balistreri et al. [4]; red solid: two distributions with Cobb-Douglas mean and standard deviation estimate from EPPA [37] and standard deviation set to 0.20.

- 3 additional elasticity of substitution parameters common across the (P) industries
- 3 additional elasticity of substitution parameters common across the (S) industries

We use uncorrelated Gaussian distributions to model the uncertainty in these elasticity of substitution values.[3] Instead of trying to combine disparate and often contradictory estimates of means and standard deviations, we centered our parameter distributions at relatively standard mean values and set the standard deviation to 20% of the mean.

A more detailed examination of the major sources of sensitivity was conducted by focusing specifically on the sensitivity to uncertainty in the 16 Armington international trade elasticities. Parameters for this study were again assumed to be the same in every region.

2.3 Statistical Methods

Approaches to quantifying model uncertainty include statistical sensitivity analysis [1, 22] and stochastic parameter control [29, 30]. The statistical approach evaluates the CGE model using random samples drawn from distributions defined around the uncertain parameters. Monte Carlo methods, for example, can be conveniently applied to sample the distributions but require significant computational power for large parameter spaces. Stochastic parameter control accounts for the temporal evolution of uncertain parameters and their relationships by incorporating estimated variance-covariance matrices.

We use the statistical approach with Monte Carlo methods to sample from the distributions of costs and elasticities described above. In particular, we constructed three large sample sets

- 10,000 samples drawn from our 1,600 uncertain base-year cost share parameters

[3]While some elasticity of substitution parameters may be correlated among similar industries, for example, we did not account for this possibility in the uncertainty distributions, even though such correlations would have an effect on the sensitivity of the variables. Establishing the existence and extent of the correlation would require a more detailed examination of the underlying covariance structures.

- 5,000 samples drawn from the 70 uncertain elasticity of substitution parameters
- 1,000 samples drawn from only the 16 uncertain Armington international trade elasticities

We used the Swift scripting system [42,43] to evaluate the CIM-EARTH CGE model for each sample in parallel. We used roughly 30,000 CPU-hours to complete these simulations with each simulation taking 0.4–1.6 hours to complete. At the peak, approximately 2,000 simultaneous processors were used. For some samples, the PATH algorithm failed to compute a solution within its default convergence tolerance. Those results were discarded, leaving 9,906 successful samples for the cost dataset, 4,978 successful samples for the elasticity dataset, and 999 successful samples for the Armington dataset.

We used bootstrap resampling [16] to explore the extent to which a smaller number of model simulations would have sufficiently characterized uncertainty in the simulated variables. We wanted to determine the subsample size that provides a statistical approximation of the full set with 95% confidence. To this end, we performed several two-sample tests between the bootstrap statistics of the subsample and the full set. Specifically, equality in the bootstrap means was assessed with a t-test; equality in the standard errors, the square root of the variance of the resampled statistic, was assessed with an F-test; and equality in the empirical probability distribution functions was assessed with the Kolmogorov-Smirnov KS-test. The p-value of these tests indicates whether to accept the null hypothesis that the two bootstrap samples are statistically equal. Thus, the p-value allows us to ascertain what subsample size replicates the full set with 95% confidence.

3 Results

We now present analysis of the results obtained from these simulations. Our objective is to understand their sensitivity to cost share and elasticity uncertainties. The variables we report are gross domestic product, aggregate CO_2 emissions, revenue for the steel and iron industries, and aggregate industrial and consumer electricity demand for specific regions and a global aggregate. To summarize the variables generated from the model simulations, we report the coefficient of variation expressed as a percentage ($\%c_v$), which provides a measure of the relative sensitivity to uncertainty. Specifically, it is the ratio of the sample standard deviation to the sample mean. Larger values for the coefficient of variation indicate greater model sensitivity.

When comparing values of a particular variable taken at initial and terminal years, we report the sample correlation coefficient r, which measures how much of the uncertainty can be explained by a linear regression model. The percentage of the uncertainty that can be explained by the linear regression is $100r^2$. Thus, for high correlation coefficients, $r > 0.7$, the uncertainty in the variable at the base year explains approximately half of the uncertainty in the terminal year. We also report the slope β of the sample linear least-squares regression model. To a first order approximation, the slope describes the overall rate at which the distribution of the variable changes from the base year to the terminal year.

3.1 Sensitivity to Cost Share Uncertainty

Table 3 summarizes the 2004 and 2063 coefficients of variation and linear correlations over the 9,906 viable cost-share simulations for several variables. We find that global aggregates generally have smaller coefficients of variation than their regional equivalents for both the base and terminal years, suggesting that large-scale aggregates are less sensitive to uncertainty in the cost data than are regional aggregates. This observation is likely an effect of attenuation toward the sample

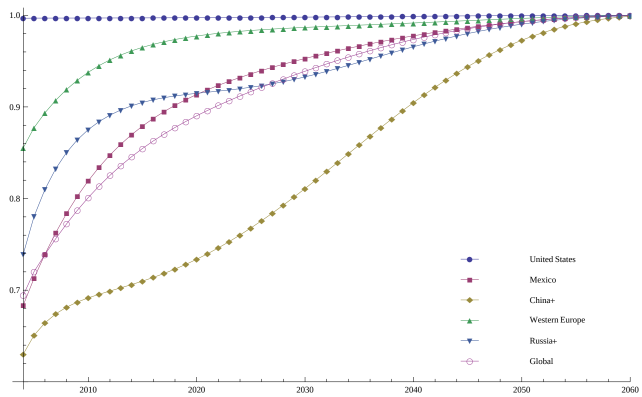

Figure 3: Correlation coefficient r between gross domestic product in the years 2004 to 2060 and the terminal year, 2063.

average, since aggregating to the global scale results in cancellation of regional variability. When exploring the sample correlation coefficient between 2004 and 2063, we find that the global aggregates generally have much weaker correlations than do their regional counterparts. This result is opposite the behavior of the coefficient of variation. Growth in the coefficient of variation from 2004 to 2063 is significantly smaller for the (L) regions. Moreover, the sample correlation coefficient in the (L) regions is very high for all variables. Over 70% of the uncertainty in their 2063 values is explained by the uncertainty in their 2004 values. In the (H) and (M) regions, r is generally smaller, suggesting that the uncertainty in the 2063 values is not well explained by the uncertainty in the 2004 values. The evolution of the correlation coefficient over time depicted in Figure 3 shows that regions with high initial uncertainty, such as China, have a gradual, near-linear increase as the gap between the initial and simulated years narrows. Comparatively, Western Europe and the United States quickly plateau to a relatively high correlation coefficient, indicating that we can explain much of the variability in simulated values between the early and terminal years.

We infer from these results that when the coefficient of variation is small, few simulations are necessary to characterize the model response with confidence. When correlation is large, we can predict the behavior of the trend and subsequently its uncertainty for that particular variable. Thus, in this situation, fewer simulations would be necessary to extract the same amount of information.

To explore this observation further, we generated 1,000 bootstrap resamples for sample sizes ranging from 5 to 9,900 of one of the most sensitive variables in our model, 2063 Chinese consumer demand for electricity. The bootstrap mean and its standard error are plotted against the resample size in Figure 4. The results of the two sample t-tests indicate that the bootstrap mean is statistically equal to the full sample mean for all sample sizes. That is, the p-value is greater than 0.05, and we fail to reject the null hypothesis. Thus, we can recover the mean model response with as few as 5 model simulations. The t-tests for other bootstrap statistics, including the mean standard deviation and mean coefficient of variation, suggest that at least 30 simulations are necessary to replicate the full set of outcomes. Although replicating the characteristics of the mean model response can be done with few simulations, the F-test for equality in the variances of the mean bootstrap resamples and the KS-test for equality of the empirical probability distribution functions indicate that larger sets are necessary to fully capture the effect of uncertainty in the share parameters. As shown in the lower panel of Figure 4, the p-value of the F-test is greater than 0.05 when the subsample size is approximately 8,200 and larger. The KS-test has a p-value

Table 3: Coefficient of variation, sample correlation, and linear regression coefficients for selected variables and regions over the 9,906 viable simulations performed to assess sensitivity to cost share uncertainty.

Variable	Region	$\%c_v$ 2004	$\%c_v$ 2063	r	β
Gross Domestic Product	Mexico	1.43	2.46	0.68	4.99
	China+	1.95	4.15	0.63	13.68
	Western Europe	1.16	1.80	0.86	4.95
	Russia+	1.51	2.78	0.74	10.70
	Global	0.69	0.85	0.69	4.21
Aggregate CO_2 Emissions	United States	1.41	1.96	0.88	3.33
	Mexico	3.08	4.13	0.88	3.39
	China+	2.00	3.65	0.49	5.31
	Western Europe	1.49	2.53	0.87	3.10
	Russia+	2.30	4.13	0.76	6.49
	Global	0.70	1.68	0.54	4.72
Steel and Iron Industry Revenue	United States	1.76	2.54	0.84	4.87
	Mexico	2.34	3.11	0.79	4.04
	China+	2.57	3.50	0.62	11.33
	Western Europe	1.44	3.00	0.71	3.76
	Russia+	1.61	4.68	0.64	16.92
	Global	0.94	2.14	0.50	6.90
Aggregate Industrial Electricity Demand	United States	0.44	1.30	0.79	10.32
	Mexico	1.09	2.61	0.62	5.60
	China+	0.88	3.39	0.30	17.06
	Western Europe	0.62	1.78	0.78	6.40
	Russia+	1.10	3.95	0.77	23.36
	Global	0.26	1.54	0.36	14.40
Aggregate Consumer Electricity Demand	United States	1.95	3.38	0.98	11.05
	Mexico	1.89	3.41	0.89	4.79
	China+	3.41	7.38	0.70	26.51
	Western Europe	1.76	3.26	0.99	7.29
	Russia+	2.30	4.30	0.78	12.36
	Global	0.80	2.18	0.68	13.09

greater than 0.05 at a sample size of 7,500 and larger, indicating that we require at least 75% of the simulations in order to be 95% confident that we can assume distributional equality with the full set of outcomes. Comparatively, the bootstrap results for Chinese gross domestic product suggest that approximately 6,700 simulations are required to capture the variance in the bootstrap mean of the full set. However, slightly fewer simulations are necessary to attain distributional equality with the full set. The KS-test indicates that approximately 65% of the full set are required in order to be 95% confident of distributional equality for Chinese gross domestic product (GDP). Thus our previous observation that variables with a smaller coefficient of variation require fewer simulations to characterize model response is substantiated with the bootstrap results. Smaller subsamples are require to capture the variance and distributional aspects of Chinese GDP ($\%c_v = 4.15$) versus Chinese consumer electric demand ($\%c_v = 7.38$).

We also estimated the bias in the model under uncertainty in the cost share parameters by comparing the gross domestic product averaged over the full set and the mean model forecast, where all costs and elasticities were set to their mean values. This bias proved to be minimal: at most 0.35% different from the mean model forecast. Furthermore, it remained relatively constant over time. A similar analysis for Chinese consumer electric demand also showed minimal bias of at most 0.25% in 2063 between the full set and mean sample.

We note that our perturbation of the expenditure data does not result in a balanced social accounting matrix in the usual sense of general equilibrium. In this case, the computed base-year commodity prices and allocations will not exactly reproduce the base-year expenditure data. In this sense, the 2004 statistics given in Table 3 measure the magnitude of the perturbations in the base-year prices and allocations required to return to equilibrium and gives an indication of the amount of the rebalancing that would be required to go from the perturbed expenditure data to a balanced social accounting matrix.

3.2 Sensitivity to Elasticity Uncertainty

Model response to uncertainty in the elasticity of substitution parameters differed significantly from the response to cost share uncertainty. The reasons are twofold. First, the elasticity of substitution does not play a significant role in the base year. In particular, the solution to the base-year model has unit inputs, outputs, and prices due to the choice of share parameters, and no substitutions are made. However, in subsequent years the variables are highly dependent on price fluctuations between commodities, and the elasticity of substitution parameters have a significant impact. Second, since we assume the elasticity of substitution parameters are the same across regions, attenuation through aggregating to global scales does not occur.

Since the model requires several iterations from the base year until it stabilizes with respect to the elasticities of substitution, we assess the response to uncertainty in these parameters by examining variables in 2010 and 2063, rather than 2004 and 2063 as we did in examining the share parameters. The coefficient of variation and sample correlation between 2010 and 2063 of several variables are summarized in Table 4 for 4,978 viable model simulations performed to assess sensitivity to uncertainty in the 70 elasticity of substitution parameters. Variables tend to be less sensitive to uncertainty in the elasticities of substitution in (L) regions because relative price fluctuations in these economies are less pronounced than for (H) and (M) regions.

The global gross domestic product has a coefficient of variation of only approximately 2.5% in 2063, while other global aggregates have substantially higher coefficients of variation. Global CO_2 emissions, for example, have a coefficient of variation of approximately 20% in 2063. Many factors contribute to this observed difference in sensitivity between global GDP and CO_2 emissions. One is that the gross domestic product is an aggregate of many variables and is dominated by more

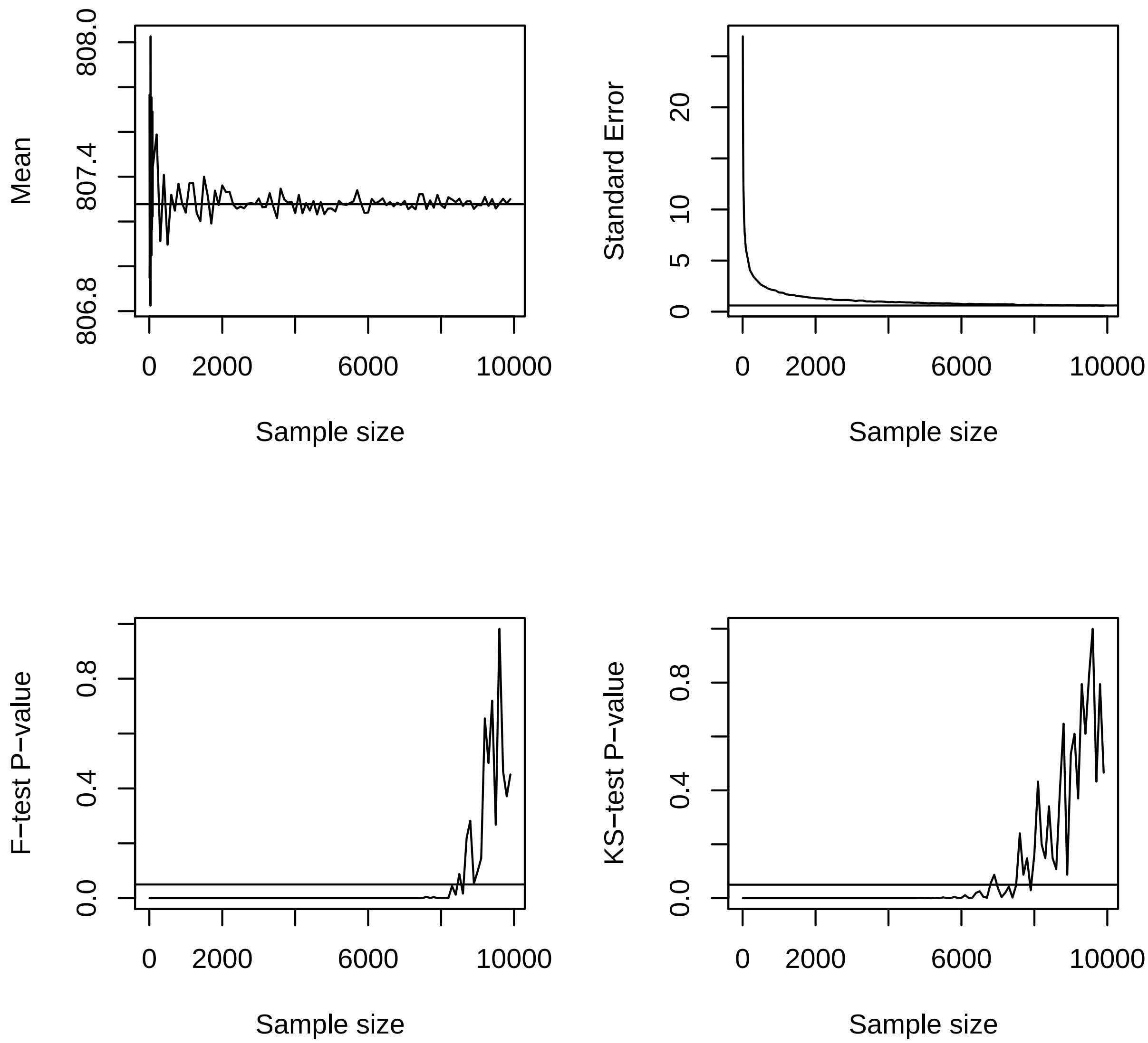

Figure 4: Bootstrap resampling of Chinese consumer demand for electricity in 2063 from the simulations examining uncertainty in share parameters. Top left: bootstrap mean versus subsample size; the horizontal line is the bootstrap mean of the full set (807.3 Mtoe). Top right: bootstrap standard error of the mean versus subsample size; the horizontal line is the bootstrap standard error of the full set (0.6 Mtoe). Bottom left: F-test p-value for equality of the variance of the bootstrap mean; the horizontal line is the 95% confidence level (0.05). Bottom right: KS-test p-value for equality of distributions of the bootstrap mean; the horizontal line is the 95% confidence level (0.05).

Table 4: Coefficient of variation, sample correlation, and linear regression coefficients for selected variables and regions over the 4,978 viable simulations performed to assess sensitivity to uncertainty in the 70 elasticity of substitution parameters.

Variable	**Region**	$\%c_v$ 2010	$\%c_v$ 2063	r	β
Gross Domestic Product	United States	0.11	2.21	0.93	78.84
	Mexico	0.20	6.01	0.61	65.32
	China+	0.68	18.54	0.23	40.78
	Western Europe	0.32	4.35	0.49	21.66
	Russia+	0.64	6.60	0.33	18.60
	Global	0.14	2.57	0.60	45.50
Aggregate CO_2 Emissions	United States	0.39	14.39	0.75	64.82
	Mexico	0.33	16.08	0.88	107.93
	China+	0.43	23.85	0.19	41.83
	Western Europe	0.50	15.79	0.64	38.69
	Russia+	1.01	23.68	0.88	72.23
	Global	0.42	17.79	0.75	92.67
Steel and Iron Industry Revenue	United States	0.19	7.56	0.69	88.37
	Mexico	0.34	11.38	0.57	59.87
	China+	0.69	16.89	0.25	49.89
	Western Europe	0.77	13.19	0.69	26.98
	Russia+	3.46	10.06	0.47	8.01
	Global	0.19	12.44	0.47	140.07
Aggregate Industrial Electricity Demand	United States	0.51	4.28	0.77	23.19
	Mexico	0.24	11.43	0.82	119.96
	China+	0.66	18.59	0.12	30.80
	Western Europe	0.50	3.16	0.68	10.72
	Russia+	0.87	16.03	0.22	23.27
	Global	0.33	11.55	0.29	50.53
Aggregate Consumer Electricity Demand	United States	0.18	8.50	0.82	189.84
	Mexico	0.25	17.27	0.88	153.10
	China+	0.67	23.09	0.35	129.52
	Western Europe	0.25	6.38	0.38	33.27
	Russia+	1.30	29.22	0.94	120.76
	Global	0.40	16.33	0.94	206.01

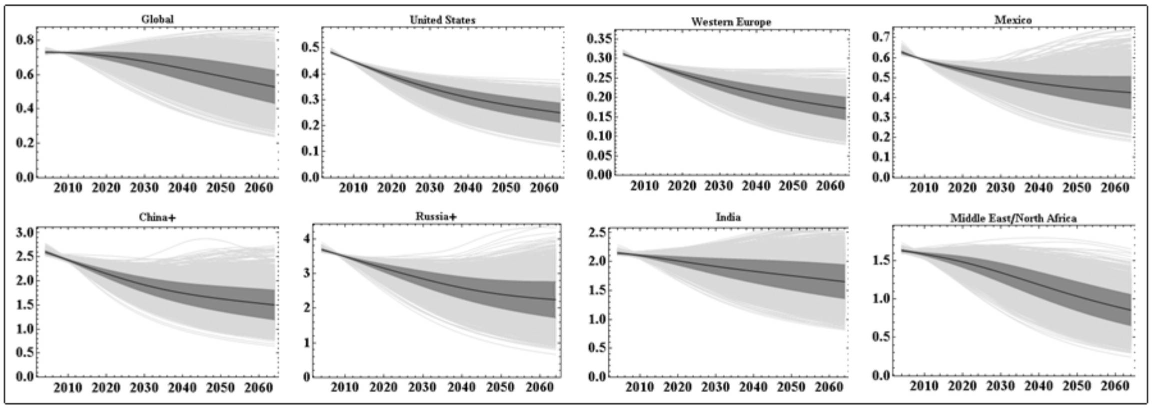

Figure 5: Carbon intensity prediction for the world and 7 of 16 model regions in kg CO_2 equivalent emissions per 2004 USD of gross domestic product.

stable economies, whereas CO_2 emissions are dominated by less stable economies. Further, the year 2063 GDP and CO_2 emissions are not correlated for the globe ($r = -0.007$) or for (L) regions such as the United States ($r = -0.054$) but have a positive correlation for (H) and (M) regions such as China ($r = 0.597$). These differences likely account for most of the discrepancy in sensitivity between these two variables.

Figure 5 shows the model predictions for global and regional carbon intensity (CO_2 emissions per unit of GDP) over for the 4,978 samples. Regional carbon intensities exhibit uncertainty largely comparable to the uncertainty in emissions themselves. In other words, we find little correlation between gross domestic product and emissions response variables for nearly all regions.

The bootstrap resamples of this set indicate that uncertainty in the elasticities of substitution results in a larger amount of variability in model outcomes. Again we examine Chinese consumer electric demand in 2063 and find that the mean depicted in Figure 6 is easily recoverable from as few as 5 simulations. The bootstrap standard error is very large (2.6 Mtoe) compared to that observed for the share parameter uncertainty set (0.6 Mtoe). The F-test for equality in variances indicates that at least 4,100 simulations were required to replicate the full set of outcomes with 95% confidence. The KS-test indicated that approximately 4,200, or 84% of the full set would be necessary to obtain the same distributional characteristics with 95% confidence. The bootstrap resamples of Chinese gross domestic product show similar results. As we found with the uncertainty in the share parameters, slightly fewer simulations are required to ensure variance and distributional equality with the full set at 95% confidence. This result is also seen in the coefficient variation, which is larger for consumer electric demand than for GDP.

For the smaller set of simulations where we examine the effect of uncertainty in the 16 Armington international trade elasticities of substitution, we allow the model to stabilize to 2020. The coefficient of variation and sample correlation between 2020 and 2063 of several variables are summarized in Table 5. Variables were most sensitive in regions and industries where international trade is essential. Relative to other regions, China displayed substantial sensitivity to uncertainty in the Armington elasticity parameters. Sizable sensitivities were also apparent for industry revenue variables that had a large revenue component from global trade, such as the steel and iron industries. While these sensitivities appear small relative to the sensitivities in our larger set of elasticity of substitution parameters, they are vital when studying the impacts of mitigation policies on international trade. The bootstrap results summarized in Figure 7 indicate that there is less

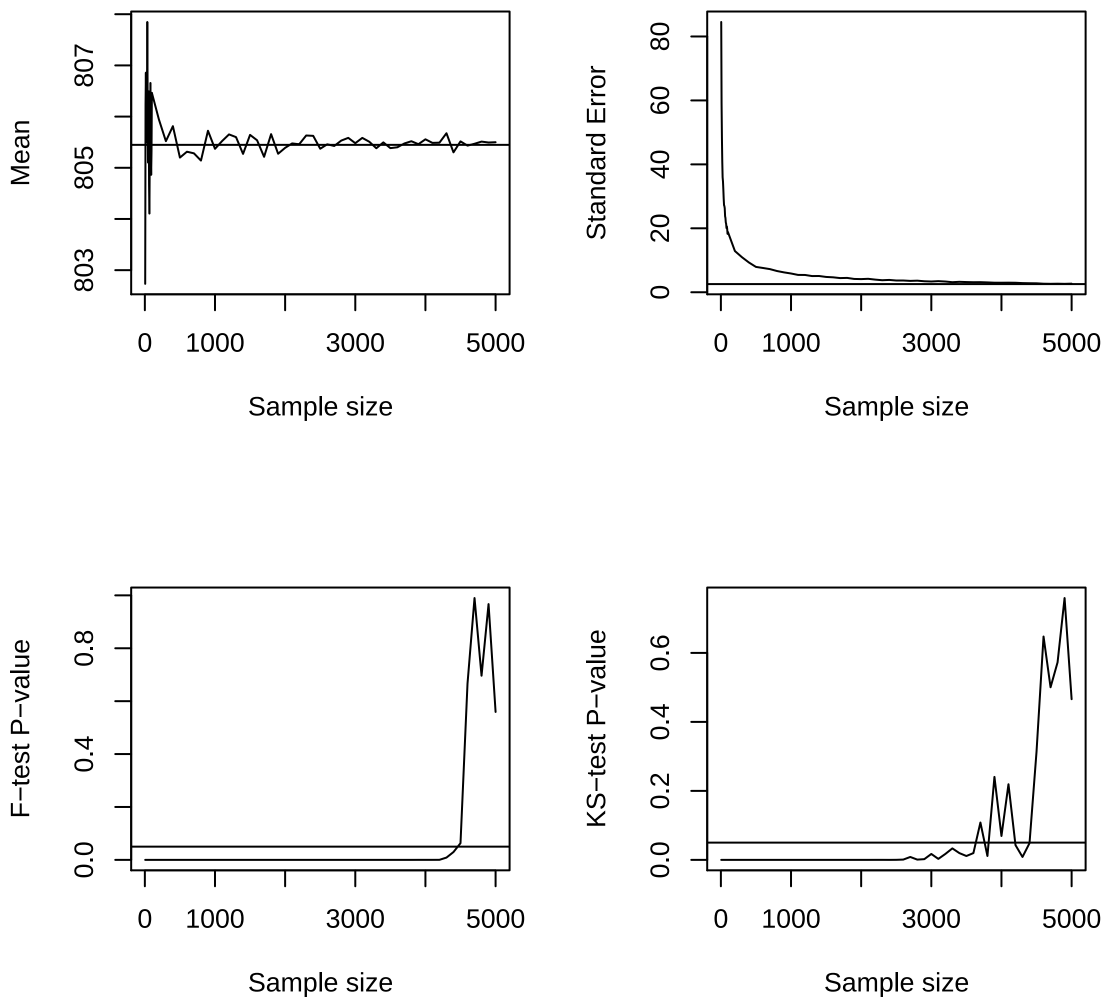

Figure 6: Bootstrap resampling of Chinese consumer demand for electricity in 2063 from simulations examining uncertainty in elasticities of substitution. Top left: bootstrap mean versus subsample size; the horizontal line is the bootstrap mean of full set (805.4 Mtoe). Top right: bootstrap standard error of the mean versus subsample size; the horizontal line is the bootstrap standard error of full set (2.6 Mtoe). Bottom left: F-test p-value for equality of the variance of the bootstrap mean; the horizontal line is the 95% confidence level (0.05). Bottom right: KS-test p-value for equality of distributions of the bootstrap mean; the horizontal line is the 95% confidence level (0.05).

variability in Chinese consumer electric demand when we introduce uncertainty in the Armington set than when we introduce uncertainty in all elasticity parameters. Again there is no statistical difference in the bootstrap means between the subsample sets and the full set. In order to capture the variance, approximately 85% of the full set is required. The KS-test indicated that we would be 95% confident that the distribution of the full set can be characterized with approximately 650 simulations.

Again, we estimated the bias in the model by comparing the gross domestic product averaged over the uncertainty sets and the mean model forecast, where all share parameters and elasticities are set to their mean values. For the 4,978 simulation set this bias remains near zero until 2020. For most regions it was less than 0.5%, but two notable exceptions are China and Africa, where the difference from the mean forecast is as much as 3% by 2063. For the 999 simulation set of Armington elasticities the bias remained near zero until 2040, and reached a maximum of only 0.4% by 2063.

4 Conclusions

We explored the sensitivity of the CIM-EARTH CGE model to uncertainty in the calibration dataset of cost share parameters and the elasticities of substitution. While our assumptions regarding these uncertainties are limited by the availability of empirical data on complexities such as parameter correlation, we incorporated information from various sources and previous studies to objectively develop distributions from which we conducted our Monte Carlo experiment. We found stark contrasts in the behavior of the model to these uncertainties, and our conclusions were highly dependent on the variable being examined. For instance, the variables exhibiting a smaller amount of initial uncertainty and higher level of aggregation displayed less sensitivity to cost share uncertainty. The coefficient of variation was substantially smaller for global gross domestic product than for Chinese consumer electric demand. Furthermore, the average global GDP over the 9,906 simulations was within 0.2% of the mean sample, indicating that few samples would be necessary to characterize the mean response of the model. This finding was confirmed with the bootstrap results, which indicated that we could replicate the mean and standard deviation of the 9,906 simulation set with 99.9% confidence with as few as 100 model simulations.

The effect of uncertainty in the elasticities of substitution was much more significant. The coefficient of variation for the 2063 Chinese consumer electric demand was 3 times larger than it was when we observed uncertainty in the cost data. We also found that the correlations between 2010 and 2063 were weaker for small-scale variables such as Chinese consumer electricity demand than their global aggregated counterparts. Furthermore, they were also weaker when we examined uncertainty in the elasticity parameters than they were for the cost share uncertainty set.

Across all three uncertainty sets, the bootstrap results suggested that we can confidently estimate the mean model response with few simulations. To capture the variability and empirical probability distribution functions, however, at least 75% of the total number of simulations we conducted were necessary. Furthermore, our observation that variables with smaller coefficients of variation require fewer simulations to characterize model response was substantiated with the bootstrap results. That is, smaller subsamples were required to capture the variance and distributional aspects of Chinese gross domestic product ($\%c_v = 4.15$) versus Chinese consumer electric demand ($\%c_v = 7.38$).

Table 5: Coefficient of variation, sample correlation, and linear regression coefficients for selected variables and regions over the 999 viable simulations performed to assess sensitivity to uncertainty in the 16 Armington international trade elasticity parameters.

		$\%c_v$			
Variable	**Region**	2020	2063	r	β
Gross Domestic Product	United States	0.01	0.15	0.90	54.83
	Mexico	0.08	0.55	0.83	14.79
	China+	0.38	2.17	0.36	7.50
	Western Europe	0.05	0.33	0.22	3.47
	Russia+	0.23	1.00	0.70	10.18
	Global	0.05	0.55	0.33	10.85
Aggregate CO_2 Emissions	United States	0.05	0.54	0.31	5.85
	Mexico	0.23	0.38	0.31	1.00
	China+	0.20	3.00	0.38	14.15
	Western Europe	0.11	0.45	0.52	3.43
	Russia+	0.16	1.27	0.17	3.18
	Global	0.20	1.03	0.53	5.80
Steel and Iron Industry Revenue	United States	0.20	1.95	0.96	21.81
	Mexico	0.09	0.55	0.94	13.12
	China+	0.63	5.21	0.49	17.93
	Western Europe	0.26	1.31	0.97	8.65
	Russia+	0.36	1.24	0.38	4.48
	Global	0.16	2.98	0.45	25.82
Aggregate Industrial Electricity Demand	United States	0.02	0.27	0.87	28.35
	Mexico	0.03	0.29	0.41	9.25
	China+	0.26	3.80	0.55	37.90
	Western Europe	0.06	0.31	0.75	7.52
	Russia+	0.30	0.95	0.40	4.21
	Global	0.10	1.48	0.43	22.01
Aggregate Consumer Electricity Demand	United States	0.02	0.21	0.52	16.93
	Mexico	0.05	0.87	0.58	18.69
	China+	0.21	3.11	0.80	64.13
	Western Europe	0.02	0.19	0.51	10.55
	Russia+	0.08	1.14	0.53	26.76
	Global	0.04	1.03	0.86	81.87

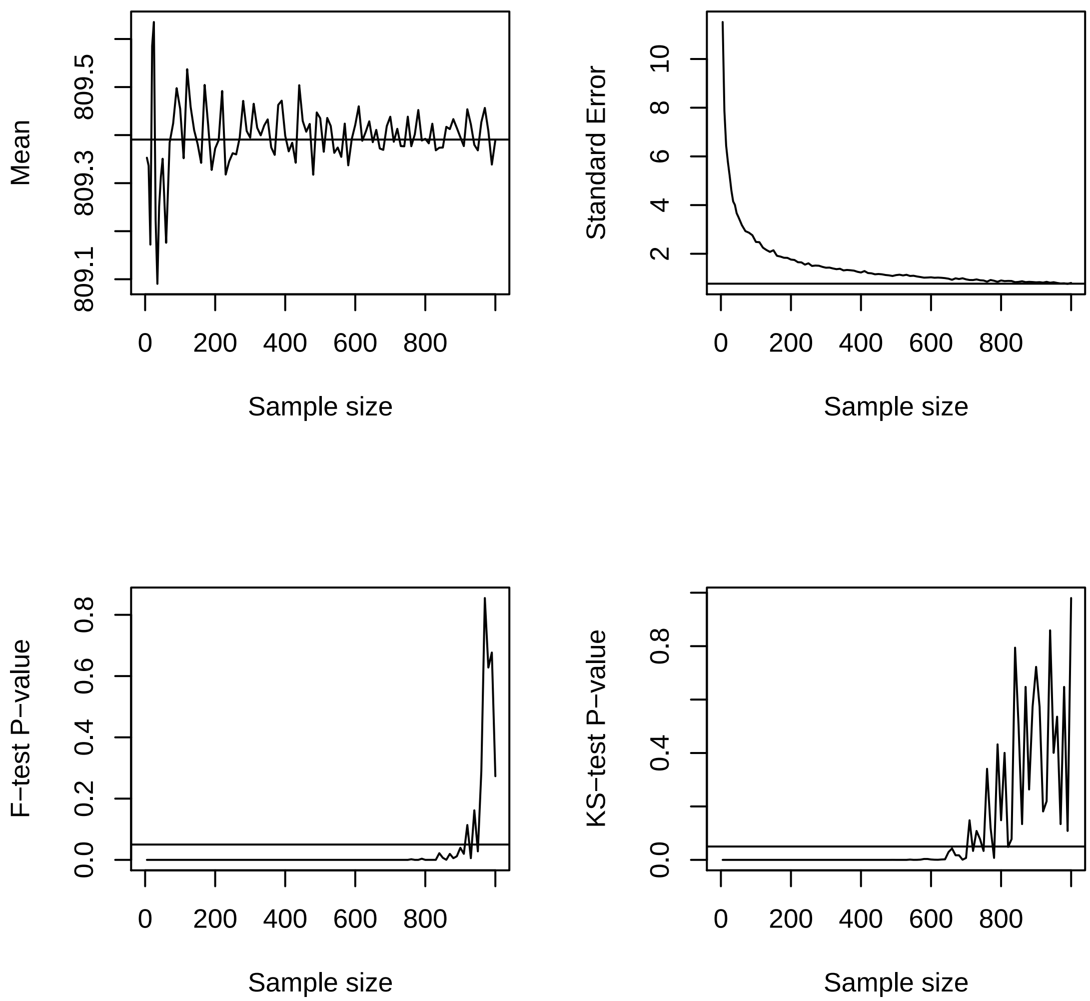

Figure 7: Bootstrap resampling of Chinese consumer demand for electricity in 2063 from simulations examining uncertainty in Armington elasticities of substitution. Top left: bootstrap mean versus subsample size; the horizontal line is the bootstrap mean of full set (809.4 Mtoe). Top right: bootstrap standard error of the mean versus subsample size; the horizontal line is the bootstrap standard error of full set (0.8 Mtoe). Bottom left: F-test p-value for equality of the variance of the bootstrap mean; the horizontal line is the 95% confidence level (0.05). Bottom right: KS-test p-value for equality of distributions of the bootstrap mean; the horizontal line is the 95% confidence level (0.05).

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A Parameter Distributions

Figure 8 compares the parameter distributions from several studies for the elasticity of substitution between capital and labor by industry. Given the sizable discrepancies in the estimates of means and standard deviations, we use Gaussian parameter distributions with relatively standard mean values and standard deviations set to 20% of the mean.

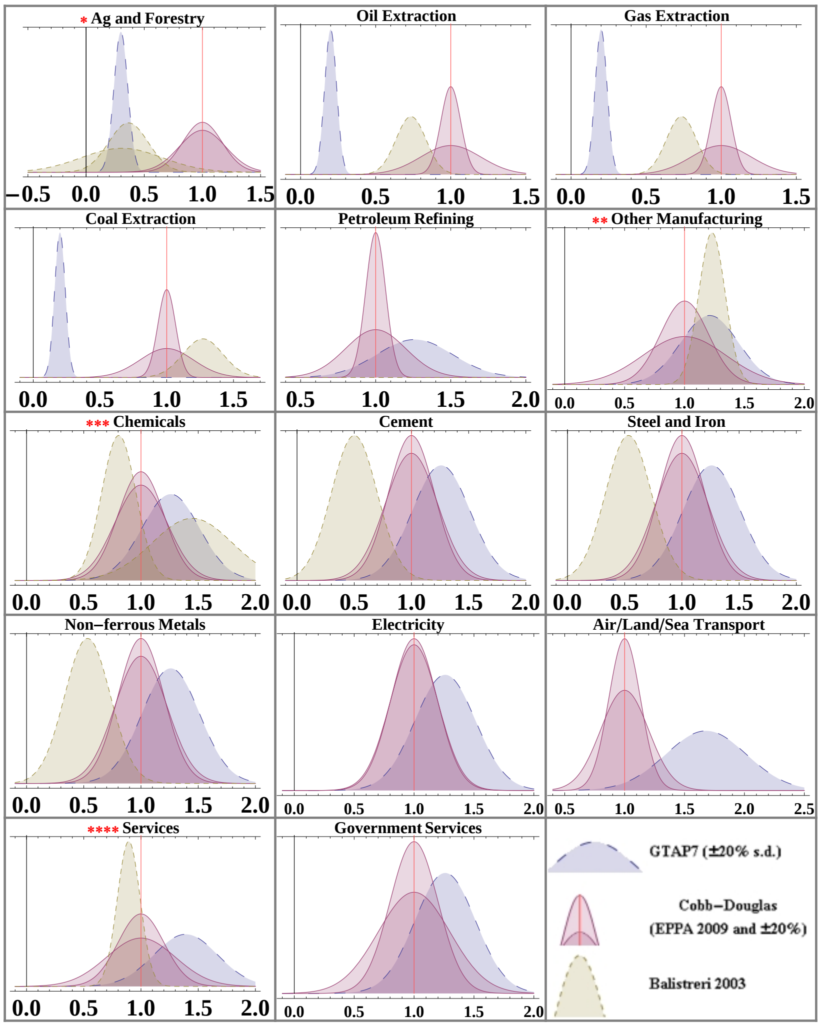

Figure 8: A comparison of parameter distributions for σ_{KL} for a variety of industries. The red line at σ=1 denotes the Cobb-Douglas point, and the black line at σ=0 denotes the Leontief or fixed-coefficients point. Some aggregate industries have multiple estimates from [4] that are relevant: * Balistreri et al. estimate two agriculture-related industries: "farms" and "agriculture and forestry services." ** Balistreri et al. estimate many industries relevant to generic manufacturing; the estimate for aggregated manufacturing and mining is shown here. *** Balistreri et al. estimate two chemical-related industries: "rubber and misc. plastic products" and "chemicals and allied products." **** Balistreri et al. estimate only one service industry: "construction services," which should not be taken as representative of aggregated services.